Technical Notes

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Collapse of Elliptic Cylinders under Uniform External Pressure

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Introduction

Recently there has been considerable interest in the behavior of elliptic cylinders under a variety of loading conditions. Yao and Jenkins[1] and Bushnell[2] have investigated the bifurcation buckling of this type of shell for the case of uniform external pressure. The theoretical results of Ref. 1 were based on a linear membrane prebuckling state, whereas the results of Ref. 2 included effects of prebuckling rotations as well. In addition to theoretical results, Yao and Jenkins presented data from tests made on 80 elliptic cylinders fabricated from polyvinyl-chloride sheet. In a number of cases there were large discrepancies between predicted and measured results. These differences and the fact that some of the experimental results showed that collapse rather than bifurcation buckling was occurring indicated the need for a type of analysis different than those used in Refs. 1 and 2. The present study was undertaken in order to investigate the nonlinear behavior of elliptic cylinders subjected to uniform external pressure and to determine the effects of initial imperfections on the collapse pressure.

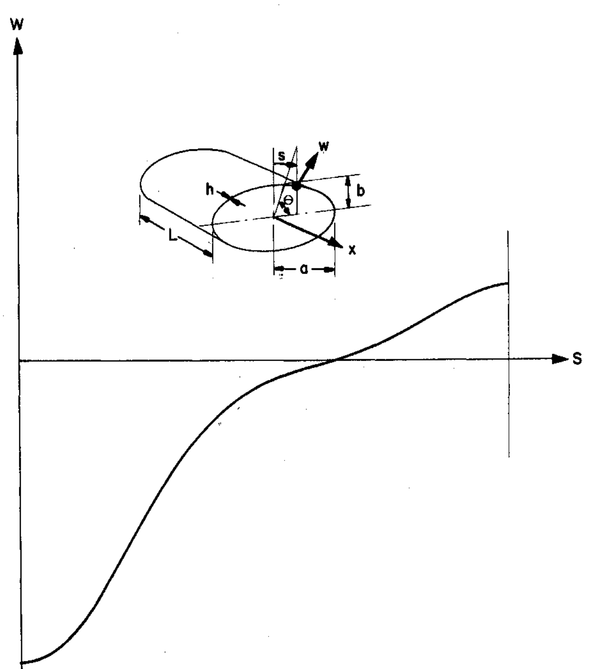

Fig. 1 Typical normal displacement at center of cylinder ($x = L/2$).

Table 1 Collapse pressures for elliptic cylinders under uniform external pressure with $a = 4.0$ in., $b = 2.0$ in., $E = 470{,}000$ psi, and $\nu = 0.37$

		Collapse pressure, psi			
Thickness h, in.	Length L, in.	Bushnell[2]	Yao & Jenkins[1]	STAGS	Test[1]
0.029	4	1.11	1.00	0.99	0.877
0.029	6	0.739	0.661	0.67	0.665
0.029	10	0.437	0.390	0.40	0.411
0.029	20	...	...	0.15	...
0.049	10	1.63[a]	1.63[a]	1.34	1.10
0.091	10	5.87	8.23	5.4	4.46

[a] $h = 0.051$ in.

Method of Analysis

The nonlinear finite difference computer program (STAGS) described in Refs. 3 and 4 was used to obtain numerical results. A brief summary of the solution procedure used in STAGS is given below.

The shell surface is covered with a mesh consisting of the coordinate lines. The unknowns are chosen as the normal displacements at the grid points and the two tangential displacements at the center of the elements defined by adjacent mesh lines. The potential energy‡ is then formed and the first variation taken. The resulting nonlinear equations are solved using a modified Newton-Raphson procedure.

Since this type of shell could be expected to collapse symmetrically about both the major and minor axes and the midlength, it was sufficient to consider only that portion of the shell bounded by the lines $\theta = 0, \pi/2$ and $x = 0, L/2$, with symmetry conditions imposed at $\theta = 0, \pi/2$ and $x = L$ (see Fig. 1).

The collapse pressures were determined by plotting the applied external pressure p vs the normal deflection w at $x = L$, $\theta = \pi/2$ and noting where the load-deflection curve approached a horizontal tangent. Typical load-deflection curves are shown in Figs. 2 and 4.

Table 2 Collapse pressure for elliptic cylinder under uniform external pressure with $L = 10$ in., $h = 0.049$ in., $E = 470{,}000$ psi, and $\nu = 0.37$

Major semiaxis a, in.	Minor semiaxis b, in.	Collapse pressure p, psi
4.500	1.165	0.34
4	2	1.34
3.587	2.536	2.65
3.301	2.859	3.90
3.137	3.030	4.70

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‡ The expression for strain energy is described in Ref. 3. The work done by the external normal pressure was obtained by modifying the expression derived by Koiter[5] for moderately large rotations.

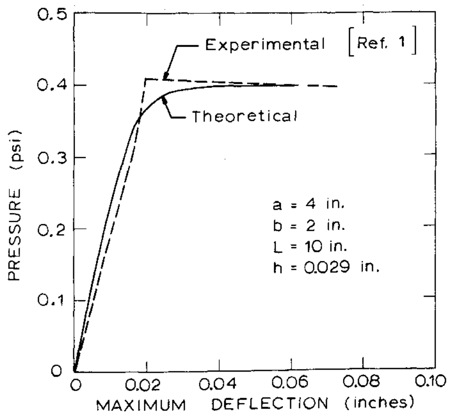

Fig. 2 Load deflection curve for elliptic cylinder.

For the cases considered here, a uniform grid with 11 axial and 21 circumferential points was used. Identical results were obtained for finer grids.

Numerical Results

Collapse pressures were obtained for a variety of elliptic cylinders. In all cases an elliptic cylinder with both ends simply supported and one end free to move in the axial direction was subjected to a uniform external pressure acting on the sides of the cylinder. Table 1 lists the collapse pressures computed by STAGS, the corresponding bifurcation buckling pressures of Refs. 1 and 2, and the experimental results of Ref. 1 for cylinders with $a = 4$ in. and $b = 2$ in.

For the cases with $h = 0.029$ in. and $L = 6$ in., 10 in. the agreement between the present theory and experiment[1] is remarkably good. Fig. 2 shows both theoretical and experimental (Fig. 5 of Ref. 1) load-deflection curves for a cylinder with $L = 10$ in. The theoretical curve is typical of all cases considered in that it is essentially linear until the pressure reaches approximately 75% of the collapse pressure, at which point it begins to roll over and rapidly approaches a horizontal tangent. A small initial imperfection of the form $w_0 = \cos 10\,\theta \sin \pi\, x/L$ was tried to see if a collapse mode similar to those for elliptic cylinders under axial load[3] would develop. Such a mode did not develop; instead the collapse mode consisted of a deep trough in the vicinity of the minor axis apex ($\theta = \pi/2$) and a shallow trough at the major axis apex ($\theta = 0$) as shown in Fig. 1. This mode was typical of all the cases considered and was similar to those observed by Yao and Jenkins. It would appear that other mode shapes might develop only for extremely thin shells such as in the example shown in Fig. 8 of Ref. 1.

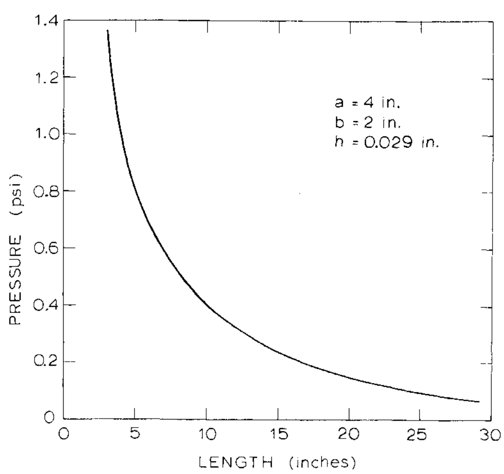

Fig. 3 Effect of length on collapse load.

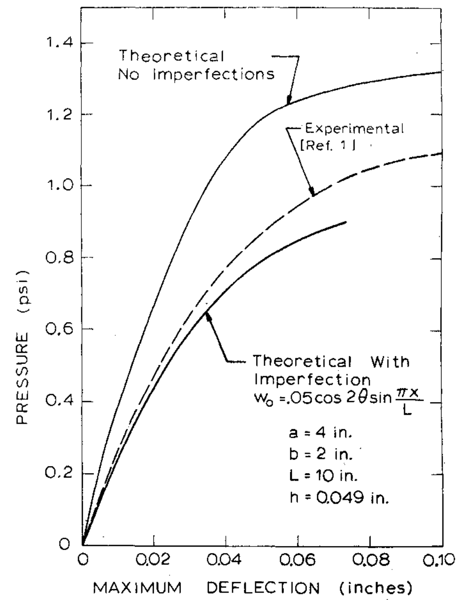

Fig. 4 Effect of initial imperfections.

Fig. 3 shows the effect of length on the collapse pressure for cylinders with $h = 0.029$ in. As could be expected, the collapse pressure is drastically reduced as the length is increased.

For those cases where the thickness is greater than 0.029 in. (and for the case where $h = 0.029$ in. and $L = 4$ in.) the theoretical collapse pressures were consistently higher than the experimental values.[1] The same phenomenon was observed by both Yao and Jenkins[1] and Bushnell[2] who accounted for it by noting that their theories allowed only for bifurcation buckling while experimental results clearly showed nonlinear behavior. However, this does not account for the discrepancies between experimental results and those predicted by STAGS. What does account for the differences is the presence of initial imperfections. This is illustrated vividly by the three load-deflection curves shown in Fig. 4 which represent the same elliptic cylinder with $a = 4$ in., $b = 2$ in., $h = 0.049$ in., and $L = 10$ in. The upper curve is for a perfect cylinder, the middle curve shows the experimental[1] result, and the lower curve is for a cylinder with an initial imperfection $w_0 = 0.05 \cos 2\theta \sin \pi x/L$. The lower curve shows a drop in the collapse load from the perfect shape of about 25% for an imperfection amplitude of only 2.5% of the minor axis length. This demonstrates a rather severe imperfection sensitivity which would certainly explain discrepancies between experiment and theory; indeed an imperfection amplitude of approximately 0.035 in. would bring the theoretical load-deflection curve into close agreement with the experimental curve.

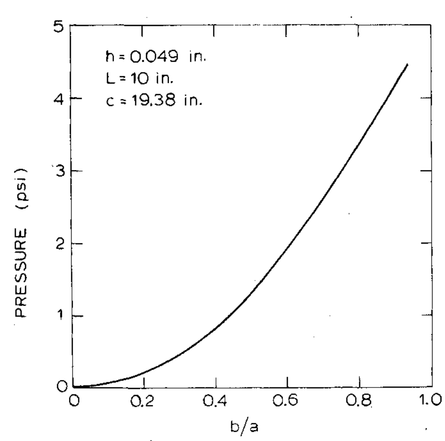

Fig. 5 Effect of b/a ratio on collapse pressure.

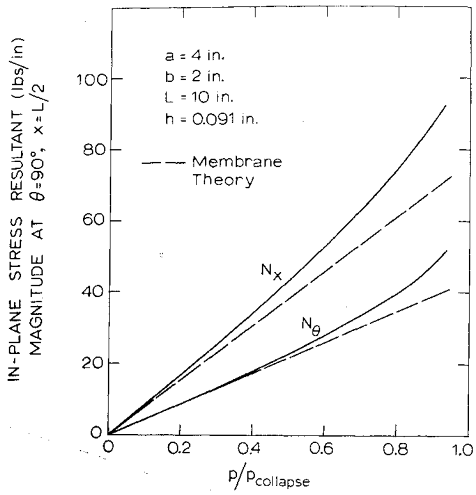

Fig. 6 Effects of nonlinearities on in-plane stress resultants.

The same imperfection sensitivity illustrated in Fig. 4 was also observed for a shell with $h = 0.029$ in. This should explain those discrepancies between theory and experiment noted in Refs. 1 and 2 for shells with thicknesses of 0.029 in. or less.

Due to the marked sensitivity of the collapse pressure to a slight change in shape for this type of shell, the effect of the b/a ratio on the collapse pressure was investigated. Table 2 shows the collapse pressures for a shell with $h = 0.049$ in., $L = 10$ in., and a constant circumference c) of 19.38 in. The results are plotted in Fig. 5. Note that for $a = 4.05$ in. and $b = 1.95$ in., the collapse pressure is higher than that for the imperfect shell of Fig. 4. This is due to the fact that the results of Fig. 5 are based on an initially straight sided cylinder while the sides of the imperfect shell of Fig. 4 are initially curved.

One reason for the difference between bifurcation buckling and nonlinear collapse results can be explained with the aid of Fig. 6 which compares the in-plane stress resultants at the minor axis apex of linear membrane theory with those computed by STAGS for a cylinder with a thickness of 0.091 in. The plot for a cylinder with a thickness of 0.029 in. is very similar except that there is very little difference in the N_θ values. The axial stress resultant (N_x) computed by STAGS becomes significantly greater than the membrane N_x for both cases as the pressure approaches the collapse pressure. However, for the hoop stress resultants (N_θ) the difference near collapse is only about 7% for $h = 0.029$ in. while it is about 26% for $h = 0.091$ in. This clearly indicates that the thicker shell allows a flattening effect at $\theta = \pi/2$ to occur due to its greater bending stiffness, whereas the thinner shell begins to collapse at the first sign of flattening.

A bifurcation type analysis cannot account for the flattening effect noticed for thicker shells; hence bifurcation buckling pressures will always be higher than the nonlinear collapse pressure.

It can be concluded that nonlinear effects are very important for this type of shell and that a linear bifurcation type of analysis should be used with caution for shells with a radius of curvature to thickness ratio (r/h) less than 270 ($h = 0.029$ in., $a = 4$ in., $b = 2$ in.). It would appear that for $r/h > 270$, a bifurcation buckling should give good results and for $r/h < 270$ ($h = 0.049$ in., $a = 4$ in., $b = 2$ in.) a nonlinear analysis should be used. In all cases, however, any imperfections should be accounted for.

References

[1] Yao, J. C. and Jenkins, W. C., "Buckling of Elliptic Cylinders Under Normal Pressure," *AIAA Journal*, Vol. 8, No. 1, Jan. 1970, pp. 22–27.

[2] Bushnell, D., "Stress, Buckling and Vibration of Prismatic Shells," *AIAA Journal*, Vol. 9, No. 10, Oct. 1971, pp. 2004–2013.

[3] Almroth, B. O., Brogan, F. A., and Marlowe, M. B., "Collapse Analysis for Elliptic Cones," *AIAA Journal*, Vol. 9, No. 1, Jan. 1971, pp. 32–36.

[4] Brogan, F. and Almroth, B. O., "Buckling of Cylinders with Cutouts," *AIAA Journal*, Vol. 8, No. 2, Feb. 1970, pp. 236–240.

[5] Koiter, W. T., "General Equations of Elastic Stability for Thin Shells," *Proceedings—Symposium on the Theory of Shells to Honor Lloyd Hamilton Donnell*, Univ. of Houston, Houston, Texas, 1967, pp. 187–227.

Development of a Sonic Boom Simulator with Detonable Gases

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Introduction

THE advent of supersonic aircraft to the field of commercial transportation is accompanied by several significant problems. Paramount among these problems is the generation of a distinctive pressure pattern known as sonic boom and its effects. At points far away from the aircraft the pressure pattern takes the shape of a classical N-wave.

In order to evaluate the effect of sonic booms on structures and human beings the desirability of developing a reliable, low-cost experimental technique capable of reproducing full-scale booms is apparent. One such technique is the generation of weak air-shock waves by the detonation of detonable-gas mixtures contained in thin mylar balloons. In order to develop an N-wave generating capability with balloons filled with detonable gases the balloon geometries which produce N-wave-forms must be determined. Once they are determined, an efficient experimental procedure must be devised such that field deployment is accomplished with ease and acceptable cost. These two aspects, namely, determining the family of N-wave generating balloons and detonation of said balloons, gave impetus to the reported program.

Conceptual Background

The use of distributed explosives to generate shock waves in air to simulate the pressure profiles expected in a sonic booms

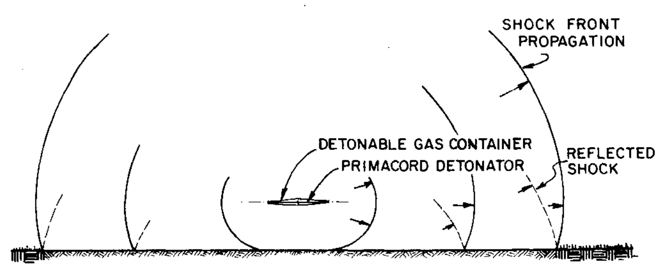

Fig. 1 Conceptual detonable gas simulation.

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